ACCOUNTING FOR OVERSPECIFICATION AND INDIFFERENCE TO VISUAL ACCURACY IN MANUSCRIPT DIAGRAMS: A TENTATIVE EXPLANATION BASED ON TRANSMISSION.

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Online appendix

In this appendix I offer the analysis of the 6 diagrams not included in the main part of the paper (part a) and some examples of unanalyzable series (part b).

Part A)

1. Ptolemy, *Almagest*, diagram 4.9

This diagram belongs to Ptolemy's *Almagest*, IV.6 [Toomer, 1988, 202, fig. 4.9]. Ptolemy finds the position of the apogee in the lunar model. K is the center of the epicycle. Λ represents the apogee and M the perigee. Point Δ is the center of the deferent. A and B are the positions of the Moon on the epicycle during two specific eclipses. Ptolemy finally finds that arc AΛ = 45;43°, while arc ΛB = 64;38°, i.e. these are non-equal arcs. In the diagram of *Vatican 180*, however, the arcs appear perfectly symmetrical. I analyzed the ratio between angles AΔΛ and ΛΔB. The starting ratio should, therefore, be 64;38°/45;43°= 1.41. In Toomer's drawing this inequality is even exaggerated. It is 2.38. I kept Toomer's exaggeration because it makes it even harder to arrive to the symmetry showed in the manuscript's diagram. The starting value is, therefore, the logarithm of 2.38 and the intended value is 0.

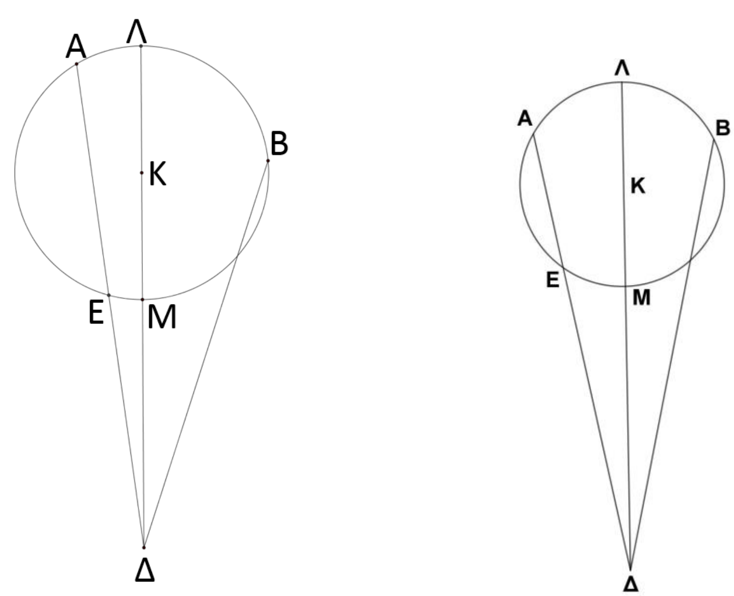
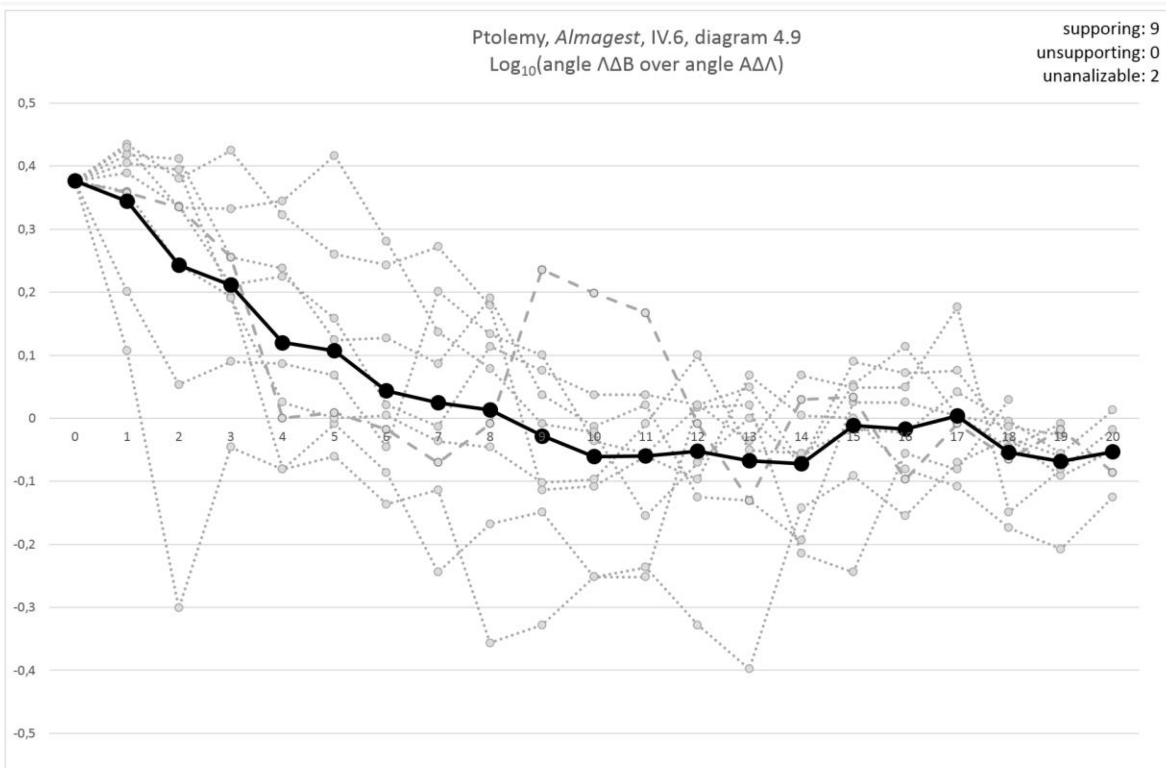


Figure 1: Diagram of the IV book of Ptolemy's Almagest. To the left, a mathematically correct diagram reproducing Figure 4.9 of [Toomer, 1988, 202]; to the right, the diagram in Vatican 180, f. 89r.

Of 11 series, 2 had to be discarderd, and the remaining 9 support my hypothesis.


Ptolemy, *Almagest*, IV.6, diagram 4.9
Log_{10}(angle ΛΔB over angle AΔΛ)
supporing: 9
unsupporting: 0
unanalizable: 2
0,5
0,4
0,3
0,2
0,1
0
-0,1
-0,2
-0,3
-0,4
-0,5
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

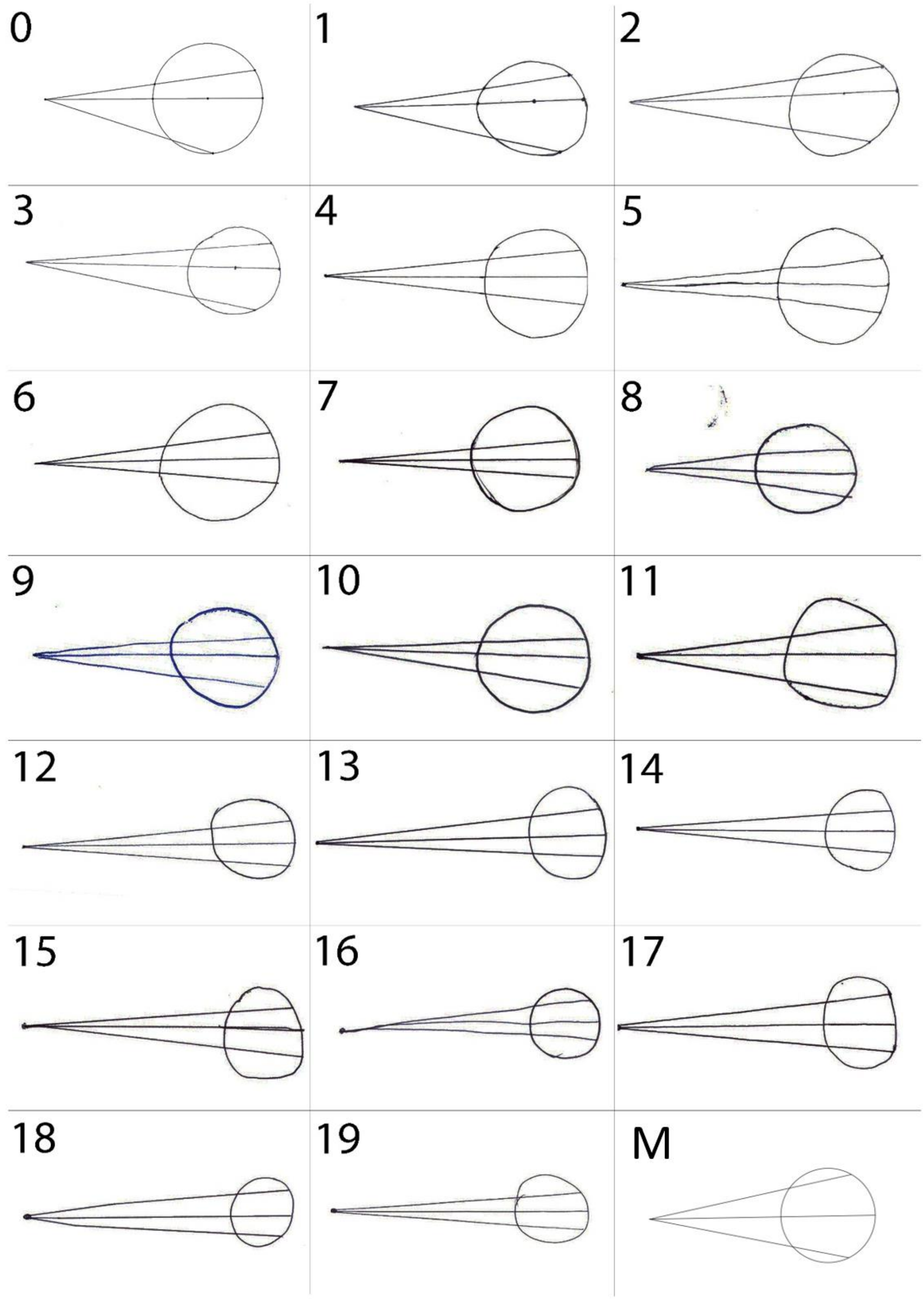


Figure 2: Reproduction of a witness series of the drawings copying the diagram of the IV book of Ptolemy's *Almagest*. Starting diagram (0): Figure 4.9 of [Toomer, 1988, 202]; final diagram (M): *Vatican 180*, f. 89r. The drawings have been rotated and resized to fit in the assigned space.

2. Euclid, *Catoptrics*, diagram 14

I analyze the diagram of proposition 14 of Euclid's *Catoptrics*. In this diagram Euclid shows that one can see the same object reflected in several plane mirrors as long as they are at a determined angle that he describes. For this description, he asks to build a penthagon with center in Θ. The eye is at B and the observed object at A. The ray from the eye will reflect in the mirror at Γ, going to Δ, then to E, and finally to A. While Θ is the center of the penthagon and, therefore, angle EΘΓ is 144° (360·2/5), in the diagram of the manuscript the angle is 180°, EΘΓ being aligned. The starting diagram was taken from [Heiberg, 1895, 7, 311]. The diagram of the manuscript was taken from *Vatican 204,* f.139r.

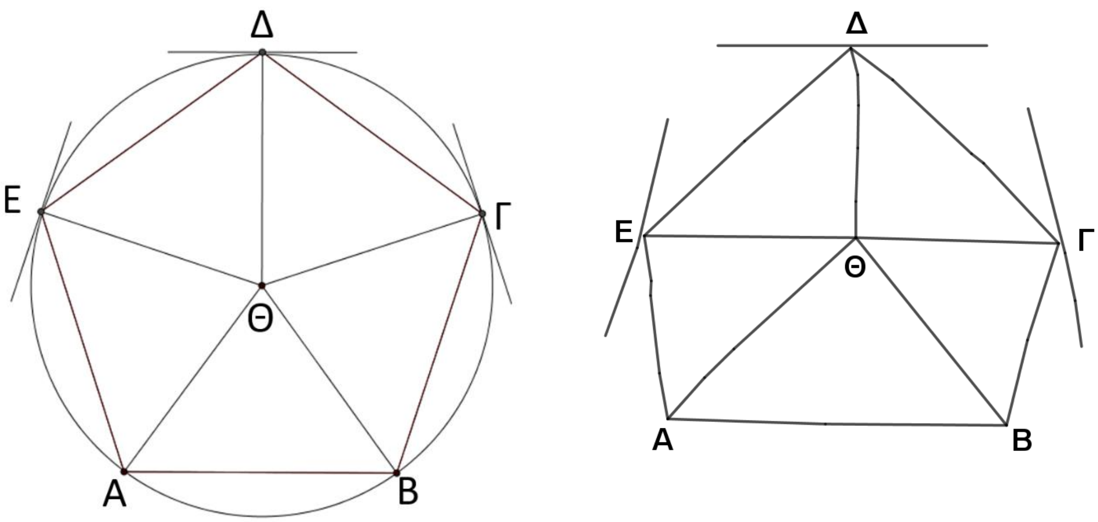
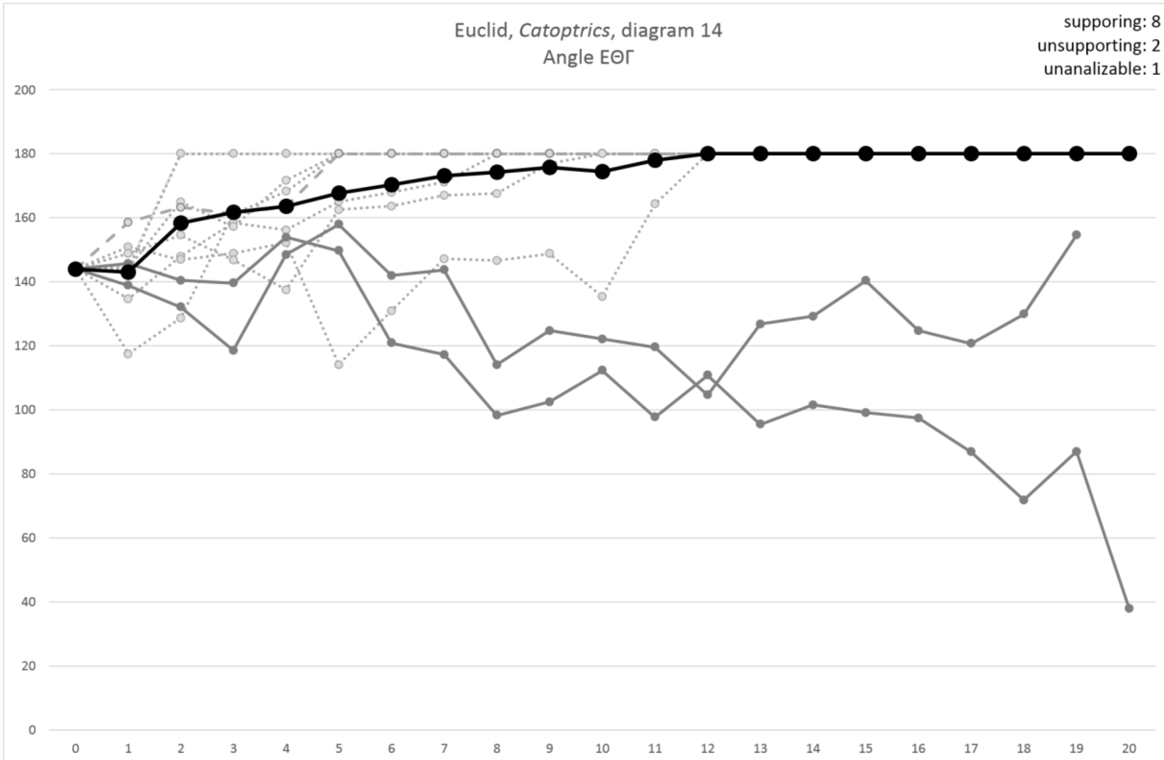


Figure 3: Diagram of proposition 14 of Euclid's *Catoptrics*. To the left, a mathematically correct diagram reproducing [Heiberg, 1895, 7, 311] with the encircling circle added; to the right, the diagram in *Vatican 204*, f. 139r. Some letters have been omitted to make the figure clearer.

From 11 series, 1 had to be discarded, 8 support my hypothesis and 2 do not. One series obtained the desired configuration at the second copy, 3 of them (including the witness series) at the fifth copy, while the rest at copy 8, 10, and 12 respectively. Once the desired configuration is obtained, it remains in the subsequent copies.



Euclid, *Catoptrics*, diagram 14
Angle ΕΘΓ
supporing: 8
unsupporting: 2
unanalizable: 1
200
180
160
140
120
100
80
60
40
20
0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

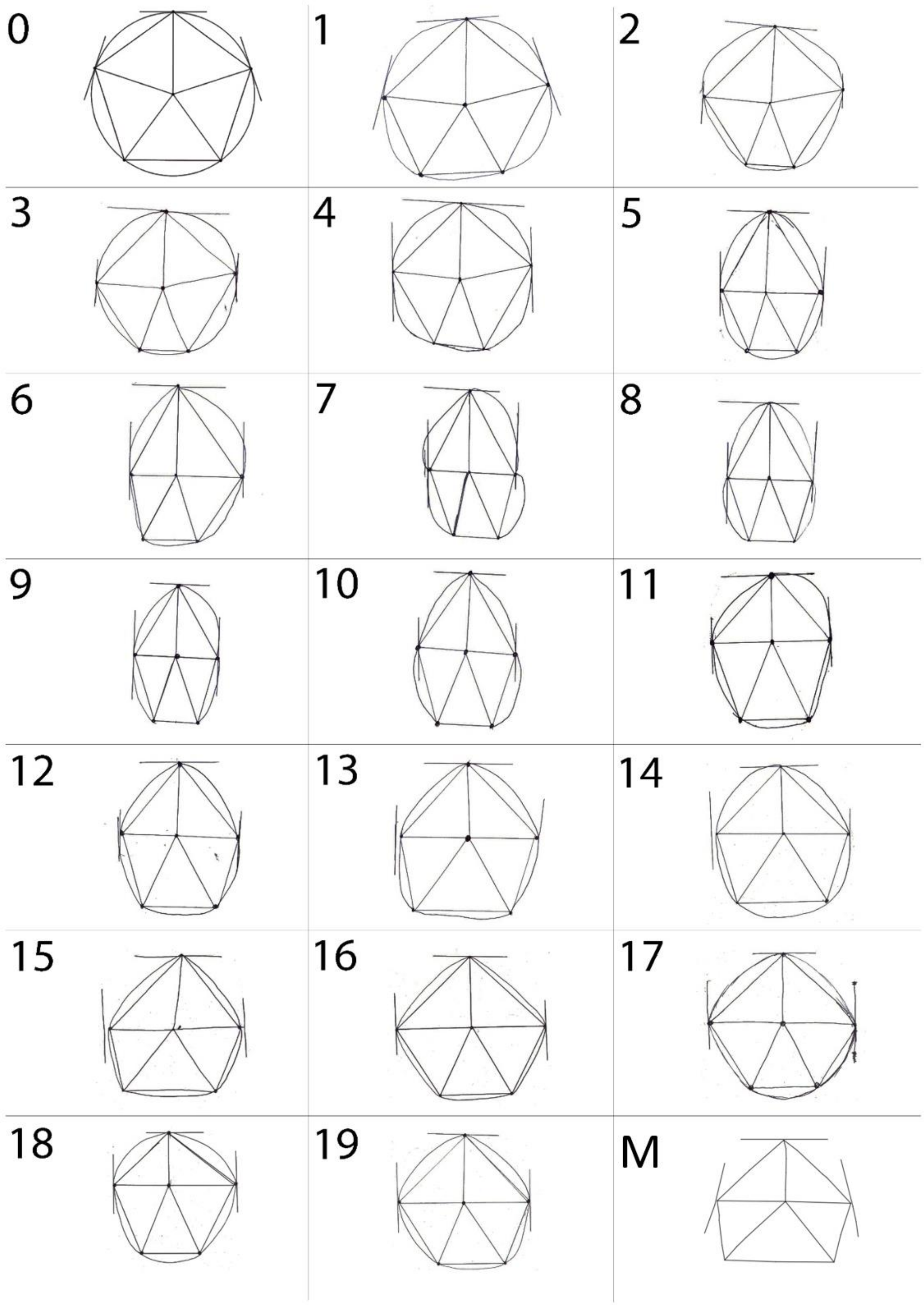


Figure 4: Reproduction of a witness series of the drawings copying the diagram of proposition 14 of Euclid's *Catoptrics*. Starting diagram (0), reproducing [Heiberg, 1895, 7, 311], with the encircling circle added; final diagram (M): *Vatican 204*. The drawings have been rotated and resized to fit in the assigned space.

3. Euclid, *Catoptrics*, diagram 15

I also analyzed the diagram of proposition 15 of Euclid's *Catoptrics*, in which Euclid proves the same conclusion as in the previous proposition, but in the cases of convex and concave mirrors. In the manuscript diagram, sides AE and BΓ of the penthagon are parallel, while they should have an angle of 36°. Both sides, also, are longer than the others, making the penthagon to looses its form and adopt a house-shape figure. In this case, Euclid doesn't explicitly ask to draw either the center, or the lines from the center to the vertices of the pentagon, so I decided to omit them contrary to what Heiberg's diagram shows. The starting figure, therefore, was taken from [Heiberg, 1895, 7, 311] but with these modifications, while the manuscript's diagram is that of *Vatican 204,* f. 139r. To establish whether sides AE and BΓ become parallel, I measured the angle between both sides. It starts at 36°, and should arrive to 0°. I did not measure the proportional length growth between sides AE and BΓ and the other three sides (i.e., if the house-shape pentagon becomes taller). In some cases this happened (like in the witness series), in some others, it didn't.

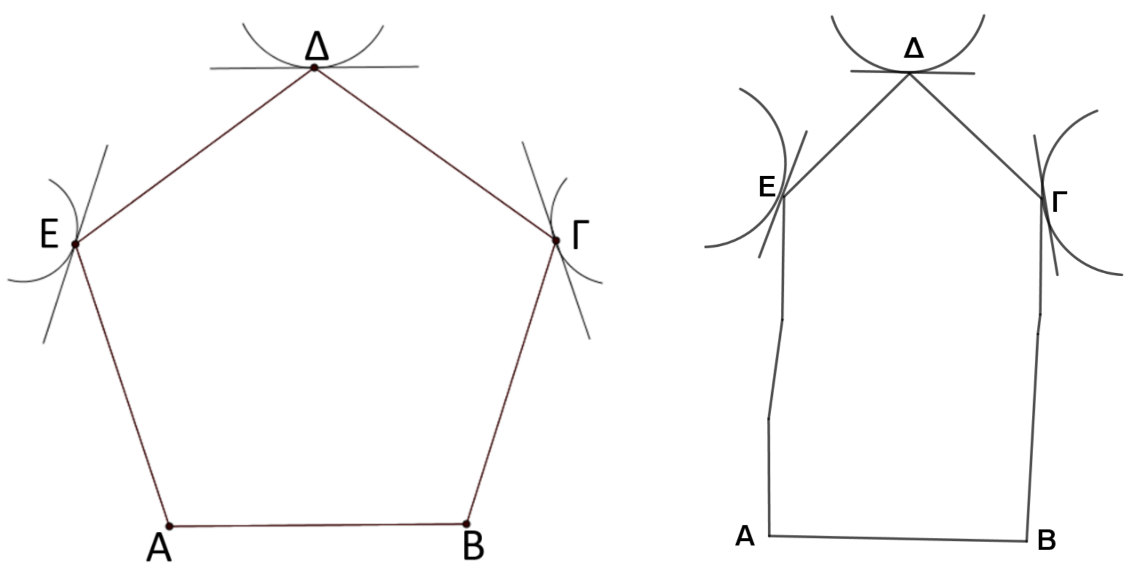
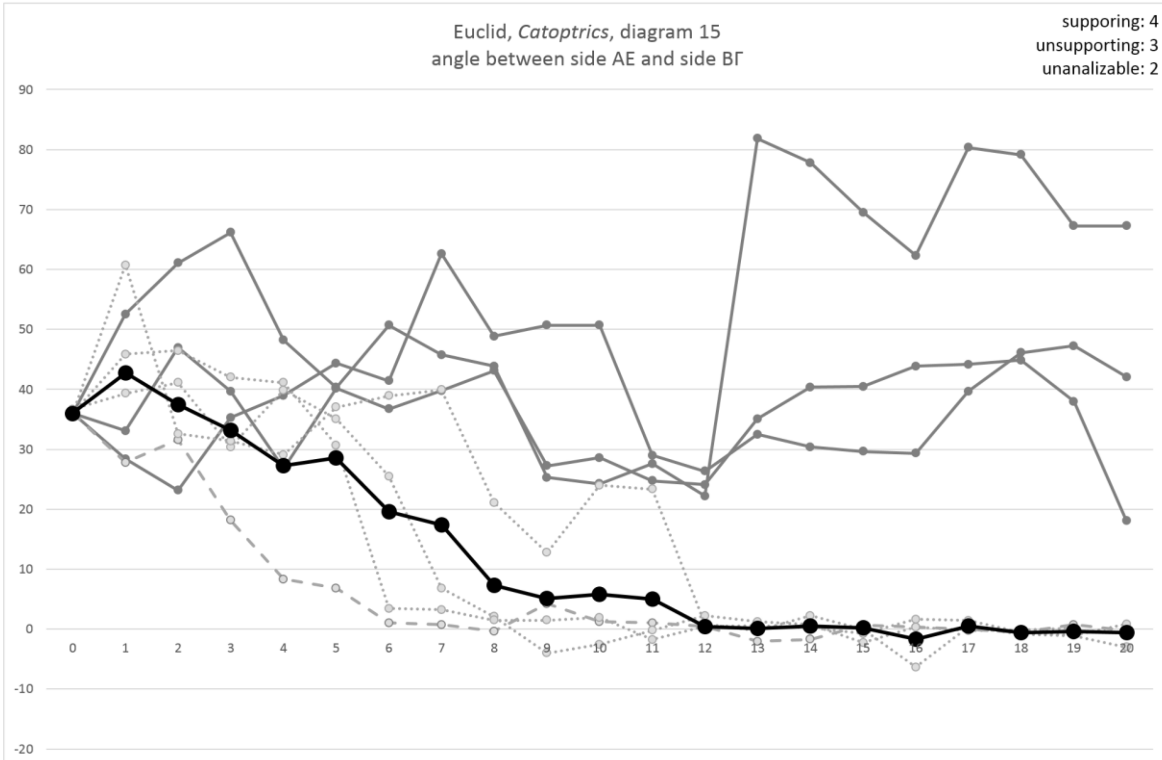


Figure 5: Diagram of proposition 15 of Euclid's *Catoptrics*. To the left, a mathematically correct diagram reproducing [Heiberg, 1895, 7, 311], but with the center of the pentagon and the lines from the vertices to the center omitted. To the right the diagram in *Vatican 204*, f. 139r. Some letters have been omitted to make the figure clearer.

In this case, the results do not support the hypothesis as much as usual. Of 9 series, 2 had to be discarded, 4 support my hypothesis and 3 do not. Around copy 12, the 4 supporting series arrived at the desired house-shape.



Euclid, *Catoptrics*, diagram 15
angle between side AE and side ΒΓ
supporing: 4
unsupporting: 3
unanalizable: 2
90
80
70
60
50
40
30
20
10
0
-10
-20
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

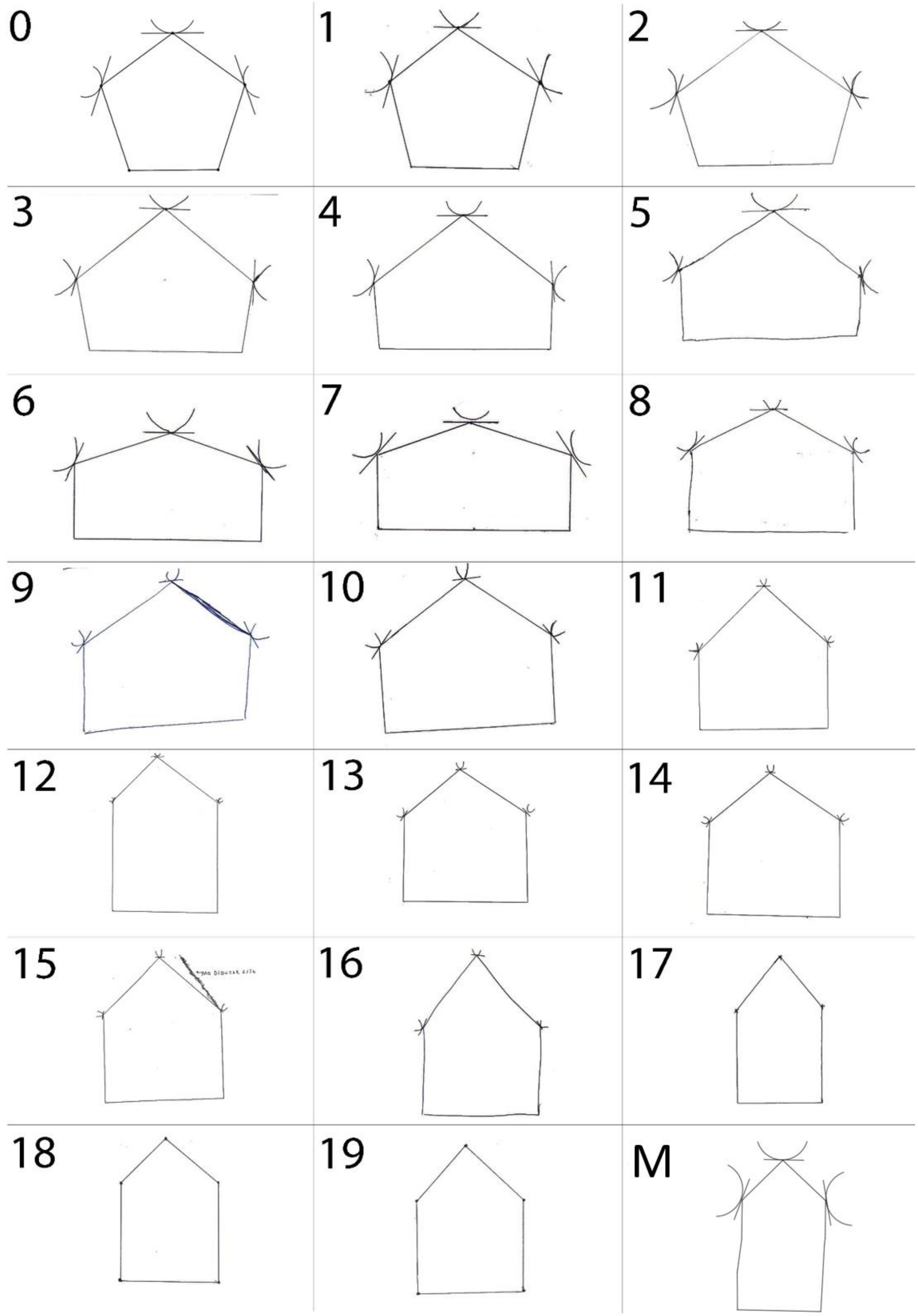


Figure 68: Reproduction of a witness series of the drawings copying the diagram of proposition 15 of Euclid's *Catoptrics*. Starting diagram (0), reproducing [Heiberg, 1895, 7, 311], but with the center of the pentagon and the lines from the vertices to the center omitted; final diagram (M): *Vatican 204*. The drawings have been rotated and resized to fit in the assigned space.

4. Euclid, *Catoptrics*, diagram 5b

At the beginning of proposition 5 of his *Catoptrics*, Euclid recalls that all visual rays reflected in a concave mirror return to the same point, adding a diagram to illustrate this phenomenon, in which arc ΑΓΔ with center in Β represents a concave mirror. He shows that the three visual rays ΒΑ, ΒΓ and ΒΔ point to Β. There is no need in the argument that arcs ΑΓ and ΓΔ be equal, as is the case in the diagram of *Vatican 204*, f. 136v. I measured the ratio between arcs ΓΔ and ΑΓ (ΓΔ/ΑΓ) and plotted the its logarithm. The starting diagram was taken from [Heiberg, 1895, 7, 295], in which the ratio between the arcs is 1.68. In the diagram of the manuscript, ray ΒΓ is considerably longer than the other two. In the series, this sometimes happens (as in the witness series) and sometimes not, but I did not analyze this characteristic.

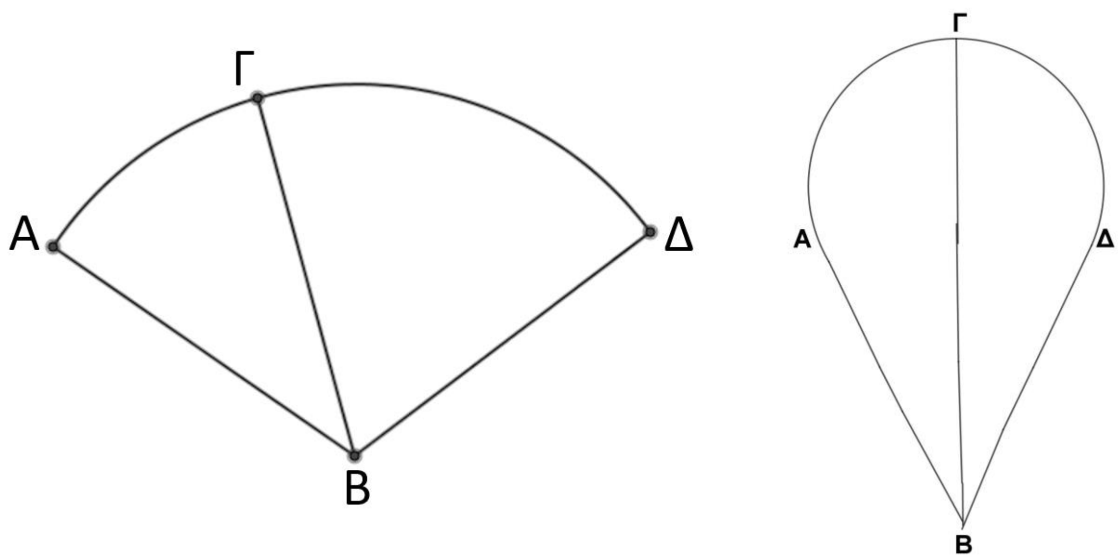
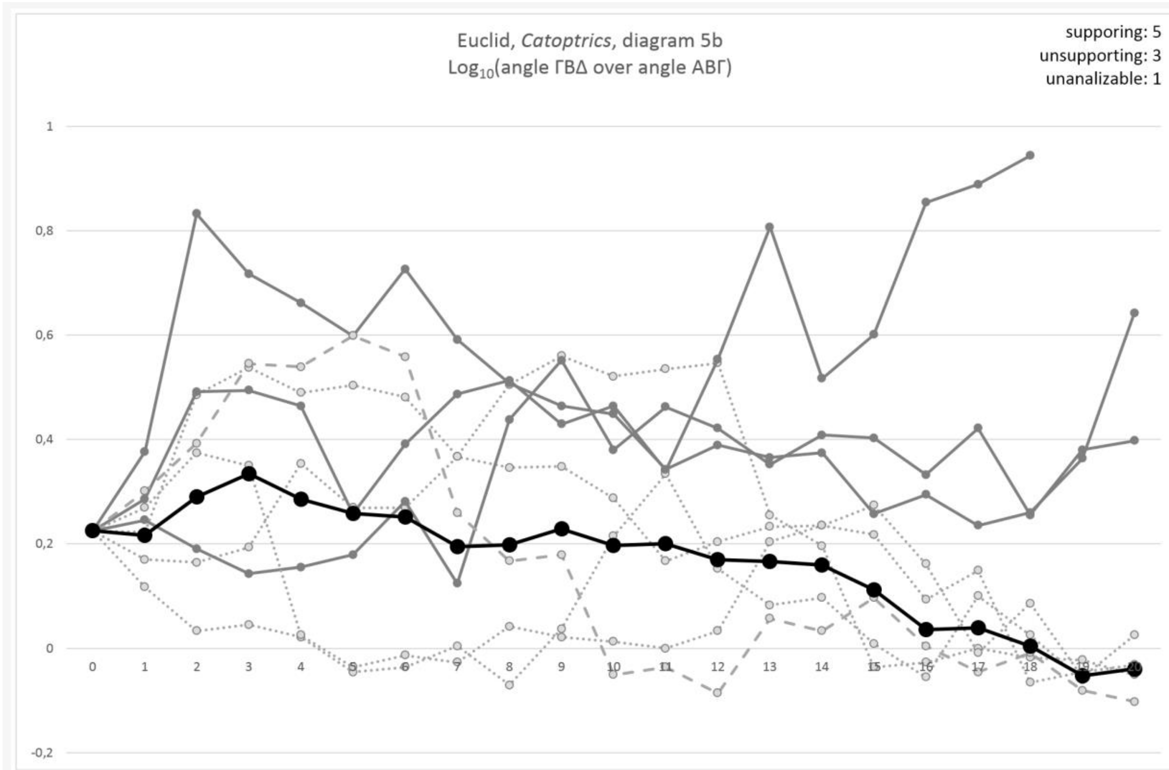


Figure 7: First diagram of proposition 5 of Euclid's Catoptrics. To the left, a mathematically correct diagram reproducing Heiberg, [1895, 7, 295]; to the right, the diagram in *Vatican 204*, f. 136v.

Of 9 series, 1 had to be discarded, 5 support my hypothesis and 3 do not.


Euclid, *Catoptrics*, diagram 5b
Log_{10}(angle ΓΒΔ over angle ΑΒΓ)
supporing: 5
unsupporting: 3
unanalizable: 1
1
0,8
0,6
0,4
0,2
0
-0,2
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

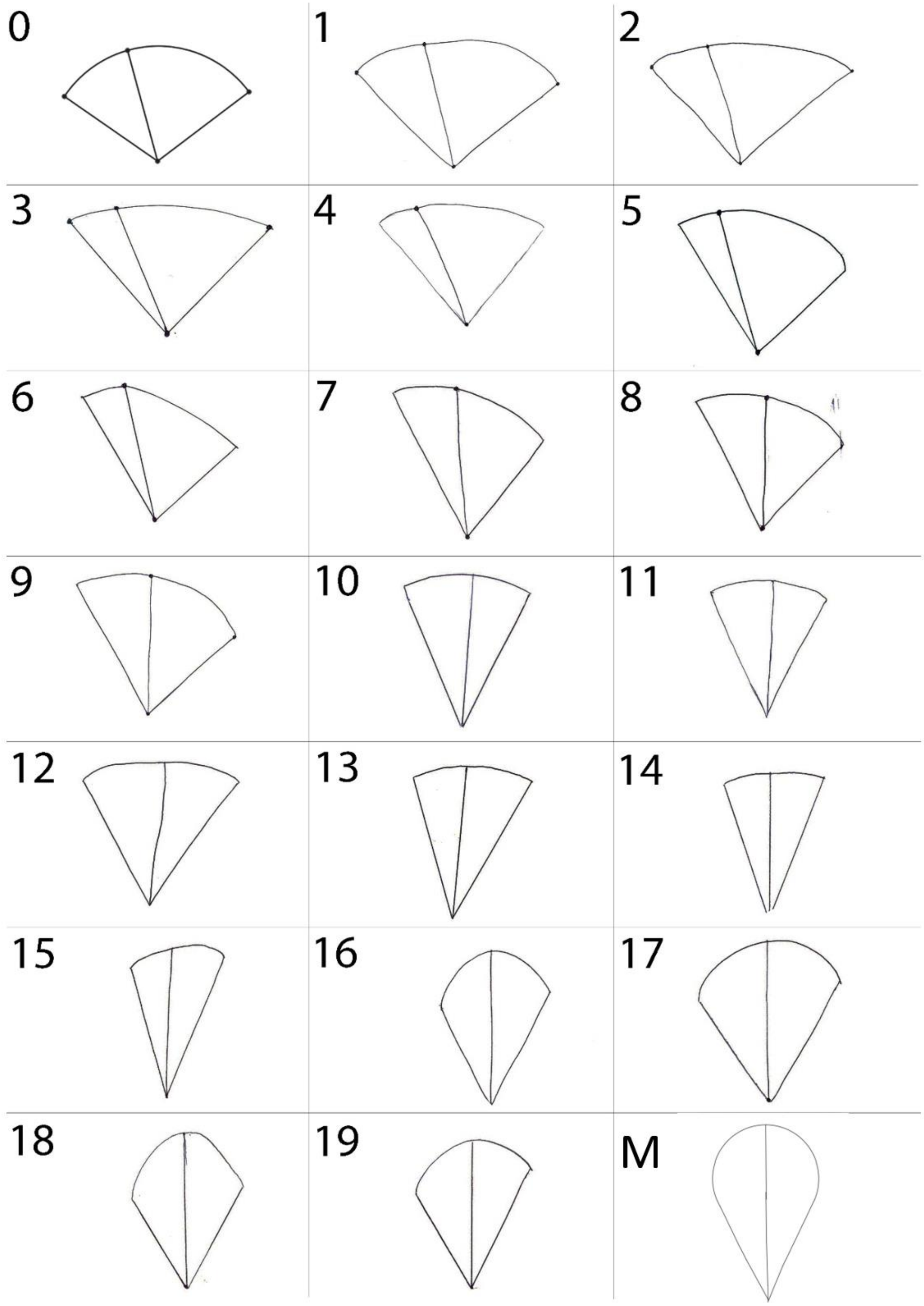


Figure 89: Reproduction of a witness series of the drawings copying the first diagram of proposition 5 of Euclid's *Catoptrics*. Starting diagram (0), reproducing [Heiberg, 1895, 7, 295]; final diagram (M): *Vatican 204*. The drawings have been rotated and resized to fit in the assigned space.

5. Euclid, *Catoptrics*, diagram 23

I analized also the diagram of proposition 23 of Euclid's *Catoptrics*, in which Euclid shows that in convex mirrors, images appear convex. He asks that arc ΑΓ be a convex mirror and the eye be at E. The ray from E to Z reflects in the mirror and comes back to E, but the other two rays EA and EΓ do not come back to E but to different points, Δ and B. Because both rays are not pointing to the center of the semicircle representing the mirror, their reflection, ΓB and AΔ should neither be parallel to each other nor to EZ. Nevertheless, in the diagram of *Vatican 204*, f. 141v, they are. They are also equidistant, which is not required. The starting diagram was taken from [Heiberg, 1895, 7, 327]. I measured the angle between AΔ and BΓ, which in Heiberg's edition is 20.69°, and should arrive at an angle of 0°.

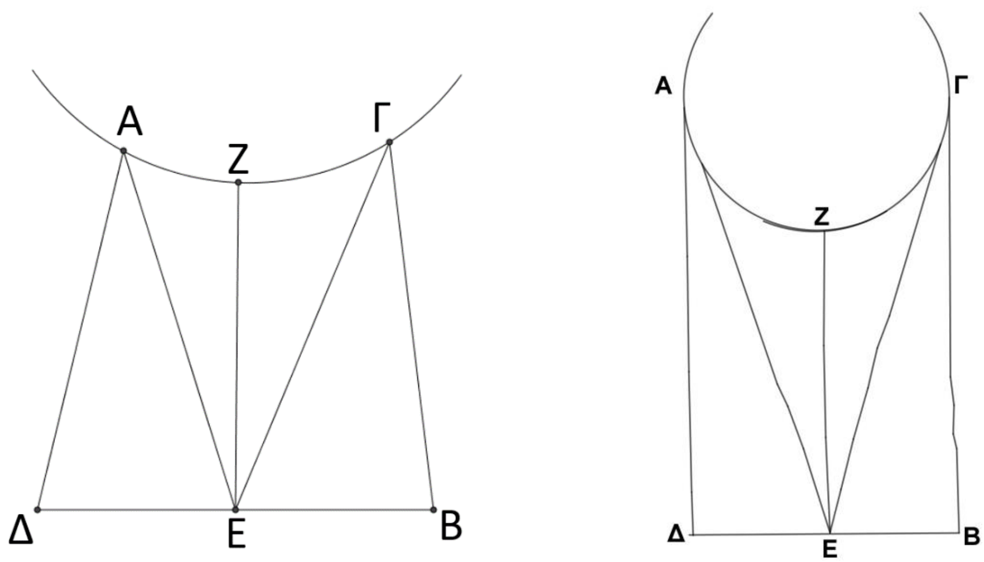
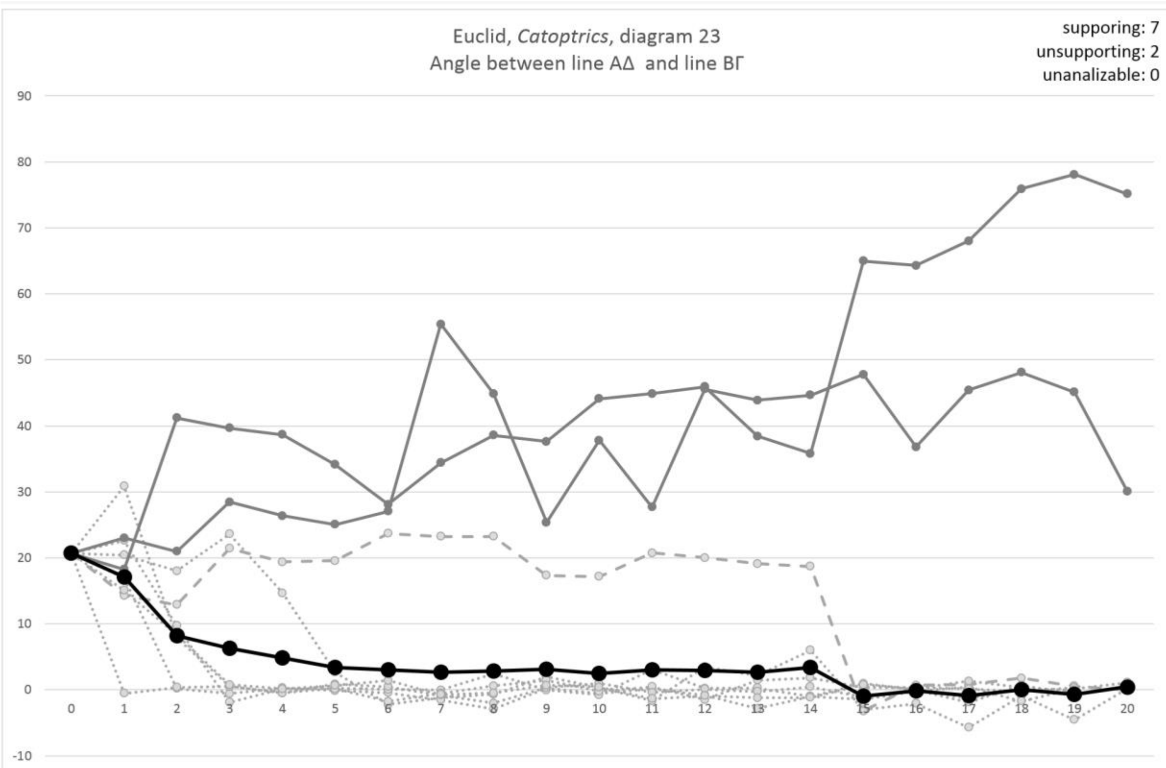


Figure 9: Diagram of proposition 23 of Euclid's *Catoptrics*. To the left, a mathematically correct diagram reproducing [Heiberg, 1895, 7, 327]; to the right the diagram in *Vatican 204*, f. 141v.

Of 9 series, 7 support my hypothesis while 2 do not. Both rays become parallel fairly soon. In one series, they become parallel in the very first copy, in another in the second, in other two in the third, etc. The one who took the most to make the transformation is the witness series, which arrives as late as at copy 15.


Euclid, *Catoptrics*, diagram 23
Angle between line AΔ and line BΓ
supporing: 7
unsupporting: 2
unanalizable: 0
90
80
70
60
50
40
30
20
10
0
-10
0
1
2
3
4
5
6
7
8
9
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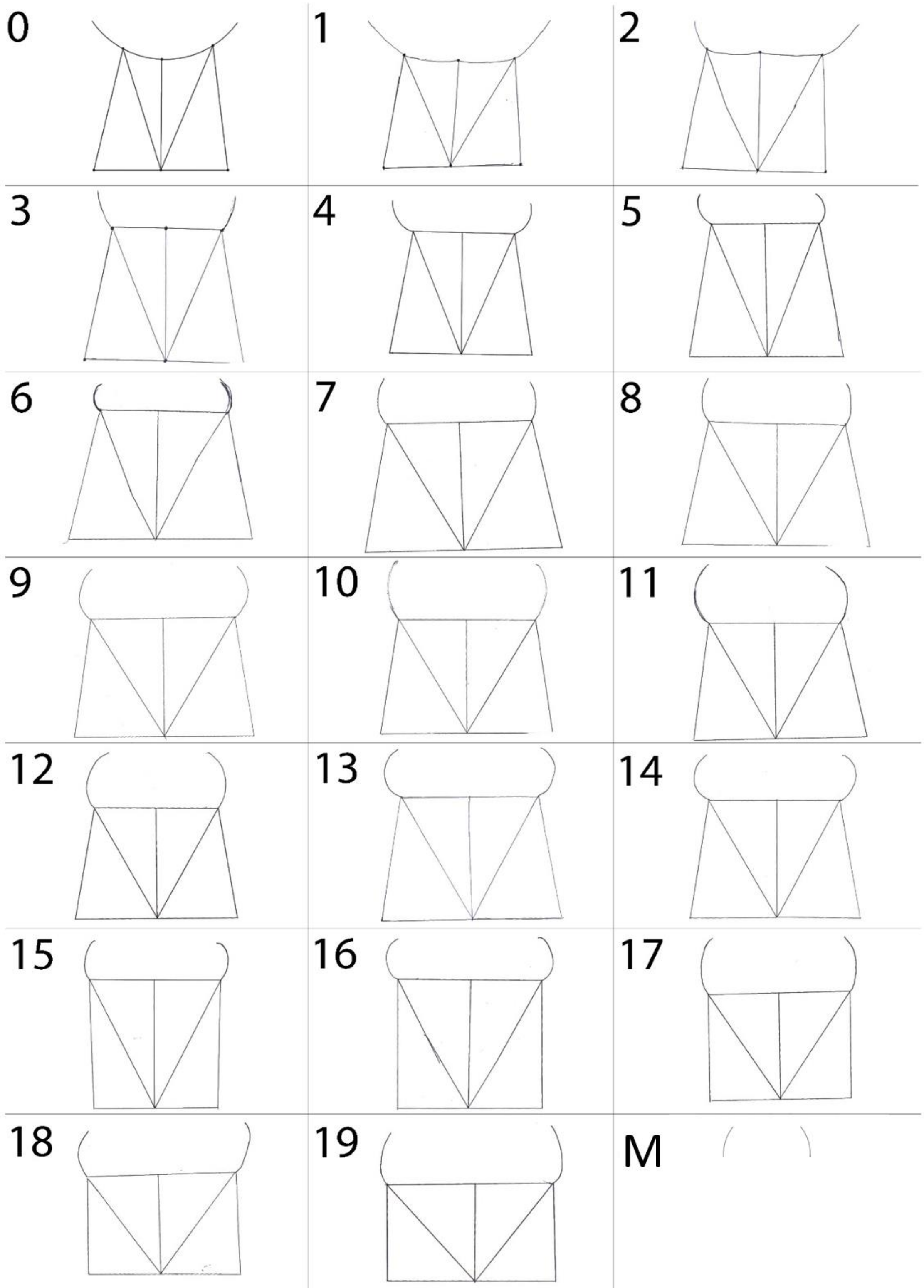
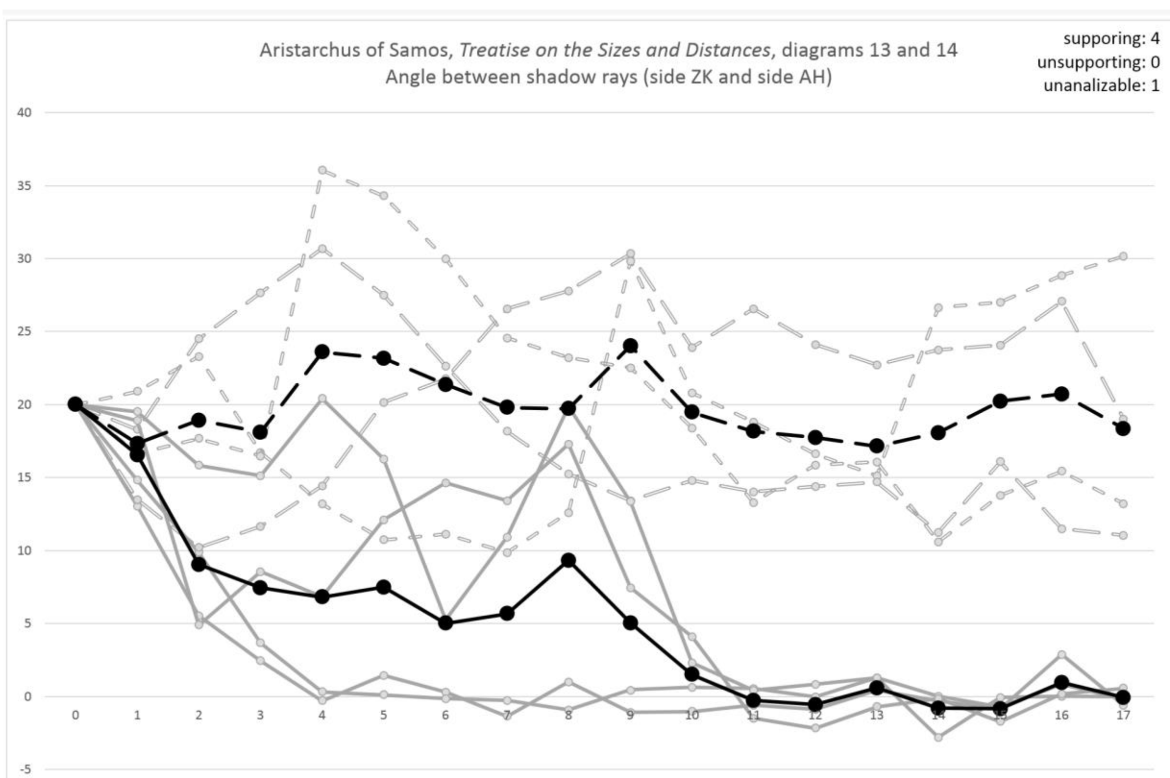


Figure 10: Reproduction of a witness series of the drawings copying the first diagram of proposition 23 of Euclid's *Catoptrics*. Starting diagram (0), reproducing [Heiberg, 1895, 7, 327]; final diagram (M): *Vatican 204*. The drawings have been rotated and resized to fit in the assigned space.

6. Aristarchus, *Treastise*, diagrams 13 and 14

Aristarchus affirms that diagrams of proposition 13 and 14 are the same, but with the moon at different locations. So, I wanted to test it: I gave the diagram of both propositions to the first student in the same sheet and asked her to copy both diagrams, and then to pass the copy. So, each student copied both diagrams in the same sheet, one after the other. The starting figures of both diagrams were done with exactly the same proportions (taken from the diagram of proposition 13 of *Vatican 104*). So, in this case, the shadow rays of both diagrams have an angle of 20°. In this experiment, I only measured the angle of the shadow-rays of both diagrams. As expected, while the rays of the diagram of proposition 13 preserve an angle, those of proposition 14 become always parallel. I tested it 5 times, 1 had to be discarded, but the other 4 support my hypothesis. I plotted with solid lines the results of diagram 14 in the chart, while in dashed lines those of diagram 13. As it is patent, while the angle between the rays in diagram 13 preserves the value around 20°, that of diagram 14 goes inevitably to 0°. In the witness series, I show some drawings of three different series (labelled α, β, γ). In the first two series, the two rays became parallel in the very first drawings, so I show only the first 6, but in the third one, the change appears around the middle of the series, so I jump from drawing 1 to 8, as to show the drawings before and after the change.



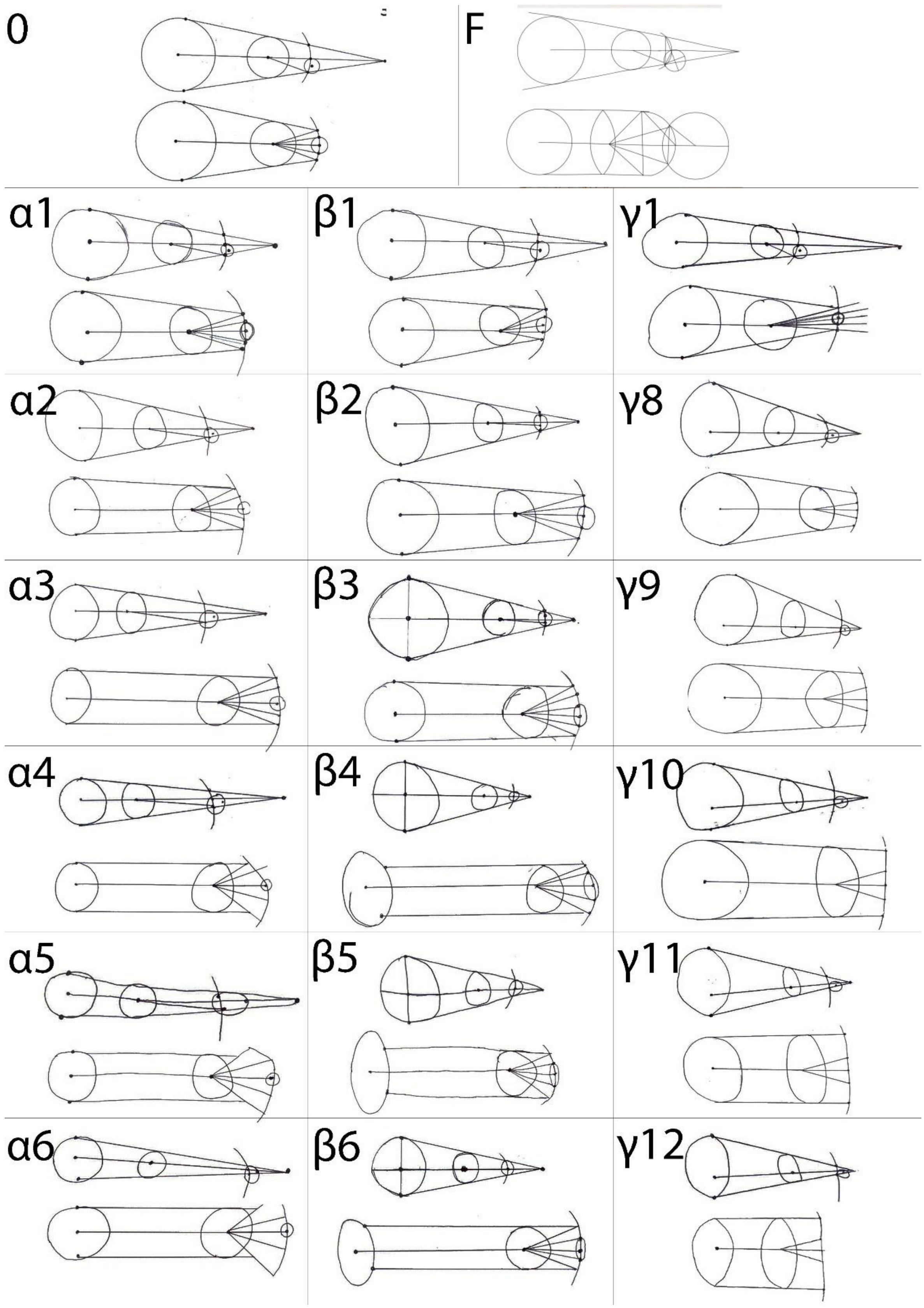


Figure 11: Reproduction of part of three witness series of the drawings copying diagrams of proposition 13 and 14 of Aristarchus's *On the Sizes and Distances of Sun and Moon*. I drew the starting diagrams (0) taking the proportions of the drawing of *Vatican 204*, the final diagrams. There are examples of three series (α, β and γ), each in each column. The number after the letter indicates the number of copy. So, in the first two series the first 6 drawings are showed, but in the third, after the first one, drawings 8 to 12 are showed. The drawings have been rotated and resized to fit in the assigned space.

Part B)

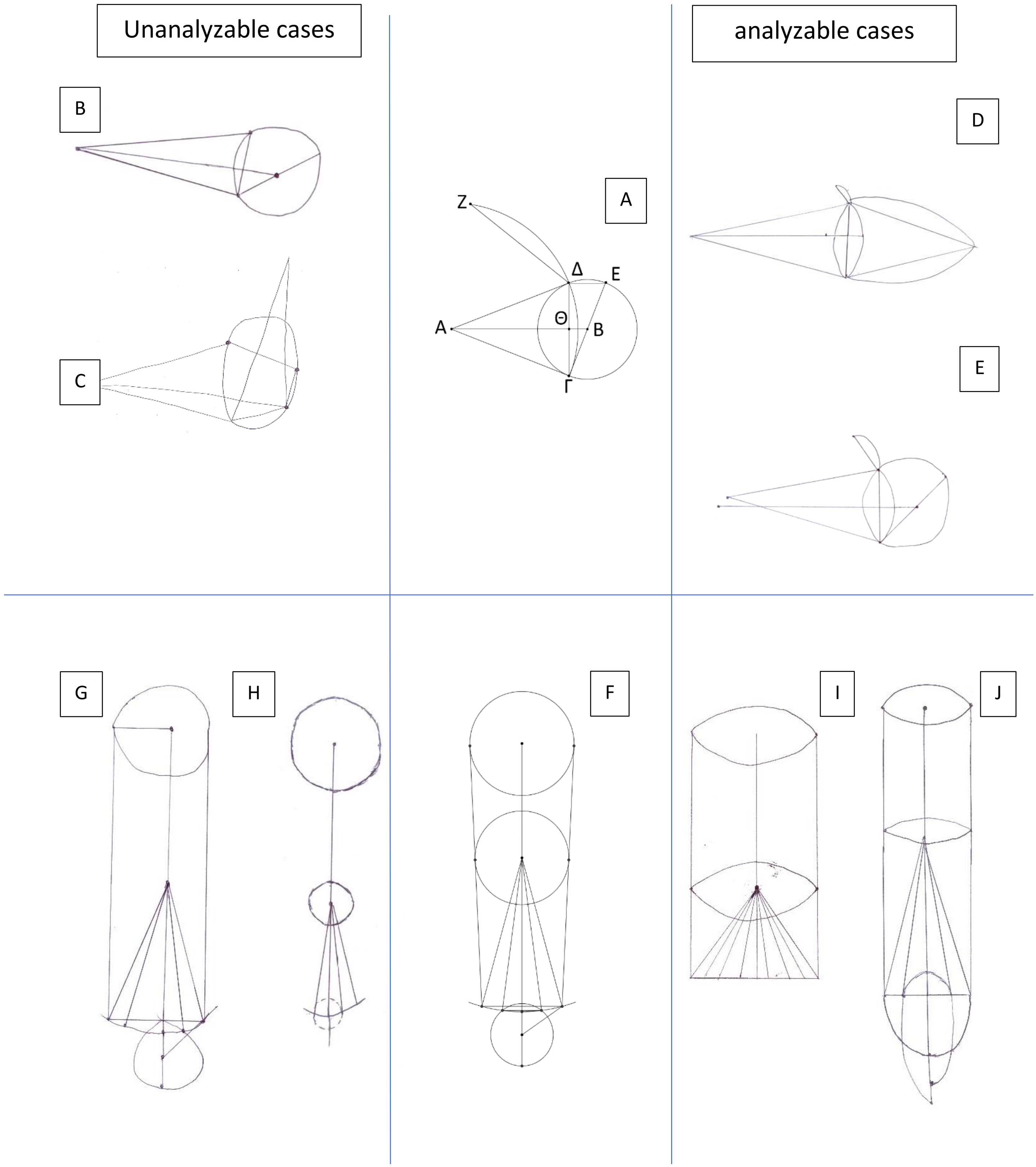


Figure 12: cases of drawings belonging to analyzable and unanalyzable series.

In Part B I show some examples of drawings belonging to analyzable and unanalyzable series. Figure 12 A is the starting diagram of proposition 11 of Aristarchus's *On Sizes* (but labeled). It belongs to the fourth case analyzed in the paper. I tested whether the proportion between lines AΔ and ΔZ, being 1 in the starting diagram, changed copy after copy. Drawings B and C are examples of unanalyzable drawings: in drawing B, line ΔZ is missing and in drawing C, these lines do not share point Δ, which is essential for their identification. In this drawing, the identification of line ΔZ is so ambiguous that it is better to discard the series altogether. Drawings D and E, however, are analyzable. In drawing D, the Moon circle (centered at B in the starting diagram) is no longer a circle; still, both lines are clearly identifiable. In drawing E, the apparent axis of the cone doesn't join the vertex of the cone; still, both lines are, again, clearly identifiable.

Figure 12 F is the starting diagram of proposition 14 of Aristarchus's *On Sizes*, already analyzed. I wanted to see whether the shadow rays became parallel and whether the Earth's circle acquired the lens-shape. Drawings G and H are unanalyzable: in drawing G, the Earth is missing, and in drawing H, the shadow rays are missing. On the contrary, drawings I and J are analyzable because both the Earth and the shadow rays are unambiguously identifiable in each of them, even if in drawing I the Moon is missing and in drawing J the Moon is no longer a circle.